

**Apportionment of Seats
to the Althingi, the Icelandic Parliament**

Analysis of the Elections on May 10, 2003 and May 12, 2007

Thorkell Helgason, PhD

The National Electoral Commission of Iceland

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Introduction

1. Short history of the election process

A Royal Decree, issued by King Christian VIII of Denmark on March 8, 1843 declared that Iceland should have its own consultative assembly, which was to be known as the Althingi. It was to be composed of 20 popularly elected representatives elected in single-member districts.

All the way up to 1959 there were gradual changes in the size of the Parliament and its election process. At the end of this period there were 52 MPs, 21 still elected in single-member districts, 12 in 6 double-member districts and finally 8 members in the Reykjavik-district. In addition there were 11 adjustment seats allocated to the parties on a national basis in an attempt to even out discrepancies in the proportionality. The candidates to fill these seats were chosen from the district list by a rather complicated method. Only those parties that had gained at least one constituency seat were eligible for the the adjustment seats. This had been so since their introduction in 1934.

These changes to the election system of the Althingi reflected the ever increasing migration from the countryside to the towns, mainly to the Reykjavik area. However, adjustment of the election act always lagged behind the migration process. This was causing two types of distortion to the composition of the Parliament. First, the rural districts were over-represented by a factor of up to ten or so! Secondly, this over-representation caused a disproportionality among the parties as a party that drew the vast majority of its votes from the farming communities was constantly over-represented.

In 1959 drastic changes in the election clauses of the Constitution were made and a new election act passed. After these changes there were only 8 multi-member constituencies with 5 or 6 members each with the exception of Reykjavik which had 12 members. In addition there were again 11 adjustment seats. Thus the total number of seats was increased from 52 to 60.

Right after these amendments, in the elections in the autumn of 1959, the over-representation of the rural districts had been reduced somewhat down to a maximum discrepancy between two constituencies of 3:1. Once more one of the parties was over-represented but only by one seat.

This act was used up to and including the elections in 1983. In the meantime the migration to the Southwest area continued. The discrepancy in the voting power was up to 4:1. And the disproportionality increased giving one of the parties on an average 2 extra seats.

The next main change in the election act came into force in the elections in 1987. The number and boundaries were not changed but some seats were transferred from the rural districts to Reykjavik and the surrounding constituency. The total number of seats was increased from 60 to 63, mainly to increase the number of adjustment seats from 11 to 13. The over-representation of the rural areas was in the beginning brought down to a ratio of 2:1 or so. But the speed of the migration increased once more so that in the elections of 1999 this ratio was once again up to the level of 4:1. However, the parties were proportionally fairly represented (at least those that passed the

eligibility requirement for the adjustment seats) in each of the four elections in which the act of 1987 was applied.

2. The new election system of 1999/2000

The provisions on elections in the Constitution were changed again in 1999, providing for a minimum of six and a maximum of seven constituencies reducing the number of constituencies from 8. Consequently the election act was amended in 2000 to provide for these changes. Constituency boundaries are determined by the act, with the exception of Reykjavik, which is divided into two constituencies; here the boundaries are determined by the National Electoral Commission (NEC) prior to each election such as to make these two parts of Reykjavik close to equal in size.

The main change was that the constituencies were made equal in size in the sense that they had the same number of constituency seats, 9 seats each. The remaining 9 seats are adjustment seats used as before to enhance proportionality in the final allocation. They three Greater-Reykjavik constituencies have 2 of these seats each; the remaining three have 1 each.

A new provision of the act requires the NEC to ensure that the discrepancy in the voting power between any two constituencies does not exceed the ratio 2:1. For this purpose the NEC has the authority to transfer one or more (constituency seat) right after each election. This act was applied for the first time in 2003 after which the NEC transferred one seat. No such change was needed after the next elections in 2007 but most likely another seat will be moved from the Northwest district to the Southwest district in the aftermath of the upcoming elections on April 25, 2009.

Votes are counted in each of the six constituencies separately. Only parties which have received at least 5% of the national vote may, however, be awarded an adjustment seat. This provision is new.

A final new provision applies to the assignment of the seats that have already been allocated to the constituency lists to their individual candidates. Ever since the introduction of list elections in 1915, lists have been nominated in ranked order by the candidature. However, the electors have always had the right to alter this order by renumbering individual candidates or to cross out those that they did not like. What has varied from time to time is the weight of these changes. In the period 1915-1959 the power of the electors in this respect was considerable. Consequently it happened twice that a candidate lost his prospective Parliament seat due to the changes made by the voters. From 1959-2000 it was rather difficult for the voters to have any influence on the rankings of the lists, apart from party primaries that were introduced in this period. With the new election act of 2000 the electors regained more or less the power they had prior to 1959.

Electors and Election Results

3. Electors and seats

The elections to the Icelandic Parliament, the *Althingi*, on May 10, 2003 and on May 12, 2007 were based on a constitutional amendments act (no. 77/1999) and a new election act (no. 24/2000) whereby the number of constituencies was reduced so that the country is now divided into six constituencies, the Northwest and Northeast, South and Southwest Districts and Reykjavik divided into two constituencies, Reykjavik North and Reykjavik South.

Figure 1 and table 1 show the number of seats in each constituency according to the election act. There is a certain flexibility in the allocation of the seats to the constituencies. Each constituency had in the 2003-elections nine constituency seats. These seats are apportioned on the basis of the results in the constituency alone. In addition, there are nine supplementary seats, so-called “adjustment seats”, which are first allocated to the parties to enhance fair proportional representation but are finally assigned to the constituencies, one or two in each of them.



Figure 1: Constituencies and seats as postulated by the 2000-act and used in the 2003-elections

The constituency boundaries are decided by law; however, the National Electoral Commission is authorized to draw the boundaries between the two Reykjavik constituencies prior to each election in such a way that the number of electors is approximately the same in both.

<i>Table 1: Number of seats as postulated by law and used in the 2003-elections</i>			
Constituencies	Constituency seats (in 2003)	Adjustment seats	Total number of seats
Northwest	9	1	10
Northeast	9	1	10
South	9	1	10
Southwest	9	2	11
Reykjavik South	9	2	11
Reykjavik North	9	2	11
Totals	54	9	63

If the number of voters on the electoral register represented by each Parliament seat, including adjustment seats, in any constituency is more than twice that of another constituency, the National Electoral Commission shall adjust the number of seats in the constituencies to bring this difference below this ratio. However, the number of seats must never become less than six in any constituency. The reallocation first enters into force in the elections following the one where the inequality has been identified.

Table 2 shows the number of voters on the electoral register per seat in the 2003-elections. It shows that the condition for a reallocation of the seats was fulfilled after those elections. Thus, prior to the next elections – which were held on May 12, 2007 – one constituency seat was transferred from the Northwest District to the Southwest District.

<i>Table 2: Voters on the electoral register and number of voters per seat</i>											
Constituencies	<u>Elections 2003</u>			Re-allocation	<u>Elections 2007</u>			<u>Elections 2009</u>			<u>2013</u> Reallocation (expected)
	Voters	Seats	Voters per seat		Voters	Seats	Voters per seat	Voters (estimated)	Seats	Voters per seat (est.)	
Northwest	21.247	10	2 125	-1	21 126	9	2 347	21 294	9	2 366	-1
Northeast	27.298	10	2 730		27 881	10	2 788	28 362	10	2 836	
South	28.344	10	2 834		30 592	10	3 059	32 505	10	3 251	
Southwest	42.812	11	3 892	1	54 584	12	4 549	58 203	12	4 850	1
Reykjavik North	42.761	11	3 887		43 756	11	3 978	43 784	11	3 980	
Reykjavik South	48.842	11	4 440		43 391	11	3 945	43 748	11	3 977	
Totals	211.304	63	3 354	0	221 330	63	3 513	227 896	63	3 617	0
		Maximum	4 440			Maximum	4 549		Maximum	4 850	
		Minimum	2 125			Minimum	2 347		Minimum	2 366	
		Ratio	2,090			Ratio	1,938		Ratio	2,050	

Elections are expected to be called for on April 25, 2009. Current estimate of the number of voters on the register in these elections indicates that again a seat must be

transferred from the Northwest constituency to the Southwest one, reflecting the ongoing migration from the provinces to the capital area. This is shown in table 2.

4. Election results in 2003

Six parties were represented in the 2003-elections in all constituencies. In addition there was an independent candidate in the South District. The election results are shown in table 3.

	Pro-gressive Party	Indepen-dence Party	Liberal Party	New Force	Social demo-cratic Alliance	Indepen-dent candi-date	Left-Green Move-ment	Valid votes in total	Participi-pation
<i>Party symbol:</i>	B	D	F	N	S	T	U		
Northwest	4 057	5 532	2 666	122	4 346		1 987	18 710	89.3%
Northeast	7 722	5 544	1 329	136	5 503		3 329	23 563	87.5%
South	5 934	7 307	2 188	166	7 426	844	1 167	25 032	88.9%
Southwest	6 387	16 456	2 890	399	14 029		2 671	42 832	88.5%
Reykjavik South	4 185	14 029	2 448	504	12 286		3 438	36 890	87.3%
Reykjavik North	4 199	12 833	2 002	464	13 110		3 537	36 145	85.5%
Votes in total	32 484	61 701	13 523	1 791	56 700	844	16 129	183 172	87.7%

Table 4 shows the same for the 2003-elections but in relative figures within each constituency and nationwide.

<i>Party symbol:</i>	B	D	F	N	S	T	U
Northwest	21.7%	29.6%	14.2%	0.7%	23.2%	-	10.6%
Northeast	32.8%	23.5%	5.6%	0.6%	23.4%	-	14.1%
South	23.7%	29.2%	8.7%	0.7%	29.7%	3.4%	4.7%
Southwest	14.9%	38.4%	6.7%	0.9%	32.8%	-	6.2%
Reykjavik South	11.3%	38.0%	6.6%	1.4%	33.3%	-	9.3%
Reykjavik North	11.6%	35.5%	5.5%	1.3%	36.3%	-	9.8%
National outcome	17.7%	33.7%	7.4%	1.0%	31.0%	0.5%	8.8%

5. Election results in 2007

In the elections 2007 two parties had left the scene, those labeled with N and T in the 2003-elections. A new movement, *The Icelandic Movement (I)*, candidated for the first time. Note that the party symbol of the *Left-Green Movement* was changed from U to V (at the Movement's request).

The 2007-election results are shown in tables 5 and 6. Note that the participation in the elections 2007 declined somewhat from the high participation in 2003.

<i>Table 5: Number of votes in the 2007-elections</i>								Participation
	Pro-gressive Party	Independence Party	Liberal Party	The Icelandic Movement	Social democratic Alliance	Left-Green Movement	Valid votes in total	
<i>Party symbol:</i>	B	D	F	I	S	V		
Northwest	3 362	5 199	2 432	255	3 793	2 855	17 896	86.0%
Northeast	5 726	6 522	1 378	278	4 840	4 558	23 302	84.8%
South	4 745	9 120	1 771	435	6 783	2 498	25 352	84.3%
Southwest	3 250	19 307	3 051	1 599	12 845	5 232	45 284	84.3%
Reykjavik South	2 081	13 846	2 385	1 680	10 234	5 065	35 291	82.6%
Reykjavik North	2 186	12 760	2 216	1 706	10 248	5 928	35 044	81.4%
Votes in total	21 350	66 754	13 233	5 953	48 743	26 136	182 169	83.6%

Table 6 shows the results now for the 2007-elections in relative figures within each constituency as well as nationwide.

<i>Table 6: Relative distribution of votes in the 2007-elections</i>						
<i>Party symbol:</i>	B	D	F	I	S	V
Northwest	18.8%	29.1%	13.6%	1.4%	21.2%	16.0%
Northeast	24.6%	28.0%	5.9%	1.2%	20.8%	19.6%
South	18.7%	36.0%	7.0%	1.7%	26.8%	9.9%
Southwest	7.2%	42.6%	6.7%	3.5%	28.4%	11.6%
Reykjavik South	5.9%	39.2%	6.8%	4.8%	29.0%	14.4%
Reykjavik North	6.2%	36.4%	6.3%	4.9%	29.2%	16.9%
National outcome	11.7%	36.6%	7.3%	3.3%	26.8%	14.3%

Apportionment of Seats

6. Outline of the apportionment

The apportionment of seats is carried out in three main steps. In the first step the constituency seats are apportioned on the basis of the election results in each individual constituency. In the second step the adjustment seats are allocated to the eligible parties based on the national outcome in order to achieve full proportionality between the parties irrespective of different weights of the constituencies. Furthermore, in this second step the adjustment seats are apportioned to the party lists, i.e. to the constituencies in such a way that each party gets all the seats allocated and at the same time the seats of the constituencies are filled. It is well known that such an apportionment with two sets of constraints (party and constituency constraints) are mathematically difficult to fulfill.¹

This entire process will be illustrated in this chapter by the outcome of the 2003-elections.

Having apportioned all 63 Parliament seats to the party lists the third and final step in the process is to find out which candidates from these lists take the seats. This part in particular of the process will be illustrated by the 2007-elections as the outcome then was more dramatic than the 2003-elections.

7. Apportionment of constituency seats in 2003

Constituency seats are allocated on the basis of d'Hondt's rule. In table 7 this process is shown for one of the constituencies, the Northwest District.

¹ See e.g. Thorkell Helgason and Kurt Jörnsten: Entropy of Proportional Matrix Apportionments, Norges Handelshøyskole, working paper 4/94 (1994).

<i>Table 7: Apportionment of constituency seats in the Northwest District in 2003</i>							
<i>Party symbol:</i>	B	D	F	N	S	T	U
Votes	4057	5532	2666	122	4346		1987
Votes divided by 1, 2, 3 etc.							
Outcomes of the division							
	B	D	F	N	S	T	U
Votes	4057.0	5532.0	2666.0	122.0	4346.0		1987.0
divided by 2	2028.5	2766.0	1333.0	61.0	2173.0		993.5
divided by 3	1352.3	1844.0	888.7	40.7	1448.7		662.3
divided by 4	1014.3	1383.0	666.5	30.5	1086.5		496.8
divided by 5	811.4	1106.4	533.2	24.4	869.2		397.4
divided by 6	676.2	922.0	444.3	20.3	724.3		331.2
divided by 7	579.6	790.3	380.9	17.4	620.9		283.9
divided by 8	507.1	691.5	333.3	15.3	543.3		248.4
divided by 9	450.8	614.7	296.2	13.6	482.9		220.8
Highest outcomes prior to each allocation. The maximum number in each line, being the basis for each allocation, is shown in bold.							
	B	D	F	N	S	T	U
Seat 1	4057.0	5532.0	2666.0	122.0	4346.0		1987.0
Seat 2	4057.0	2766.0	2666.0	122.0	4346.0		1987.0
Seat 3	4057.0	2766.0	2666.0	122.0	2173.0		1987.0
Seat 4	2028.5	2766.0	2666.0	122.0	2173.0		1987.0
Seat 5	2028.5	1844.0	2666.0	122.0	2173.0		1987.0
Seat 6	2028.5	1844.0	1333.0	122.0	2173.0		1987.0
Seat 7	2028.5	1844.0	1333.0	122.0	1448.7		1987.0
Seat 8	1352.3	1844.0	1333.0	122.0	1448.7		1987.0
Seat 9	1352.3	1844.0	1333.0	122.0	1448.7		993.5

The apportionment of constituency seats is shown in total in table 8.

<i>Table 8: Apportionment of the constituency seats in total in 2003</i>								
<i>Party symbol:</i>	B	D	F	N	S	T	U	Totals
Northwest	2	3	1	-	2	-	1	9
Northeast	4	2	-	-	2	-	1	9
South	2	3	1	-	3	-	-	9
Southwest	1	4	-	-	4	-	-	9
Reykjavik South	1	4	-	-	3	-	1	9
Reykjavik North	1	3	-	-	4	-	1	9
Constituency seats in total	11	19	2	-	18	-	4	54

8. Apportionment of adjustment seats in 2003

The apportionment of adjustment seats involves two substeps: First they have to be allocated to the eligible parties on the basis of the national results and secondly they must be allocated to the individual party lists in the constituencies. However, these two steps are intertwined.

Prior to this allocation process eligible parties have to be identified. Those and only those parties that receive at least 5% of the votes on a national basis are eligible to

acquire adjustment seats. The candidacies with the symbols N and T fall short of this threshold.²

The first step in the process is the allocation of the adjustment seats on the basis of the national outcome. This again is based on d'Hondt's rule. For that purpose the 54 allocated constituency seats are taken as given and the allocation of the 9 adjustment seats is continued with d'Hondt's rule.

Let us take the Progressive Party (B) as an example. They have already won 11 constituency seats. To find out the party's merits for further seats their total number of votes is divided by 11+1=12, 11+2=13, 11+3=14 etc. These quotients are called "national ranking numbers" in the election act and will be referred to under that name. These numbers in the 2003-elections are shown in table 9.

<i>Table 9: "National ranking numbers" in the 2003-elections</i>					
National ranking numbers are calculated by dividing the national outcome by the number of apportioned constituency seats plus 1, 2, 3 etc. The 9 highest figures are shown in bold.					
<i>Party symbol:</i>	B	D	F	S	U
National votes:	32 484	61 701	13 523	56 700	16 129
No. of constituency seats:	11	19	2	18	4
Highest ranking numbers	2 707	3 085	4 508	2 984	3 226
	2 499	2 938	3 381	2 835	2 688
	2 320	2 805	2 705	2 700	2 304
	2 166	2 683	2 254	2 577	2 016

The national ranking numbers are then ordered in descending order. Thus the first two adjustment seats go to the Liberal Party (F) as that party has the highest national ranking number, $13523/(2+1)=4508$.

Now these seats have to be allocated to a list of this party in one of the constituencies. The act postulates that the list with the relatively highest standing in a (theoretical) continuation of the apportionment of constituency seats shall receive this seat. Thus in each constituency the number of votes cast for each party list is divided by the number of its constituency seats that the list has already received plus 1. This outcome is then divided by the total number of eligible votes in the respective constituency. These relative outcomes are shown for all the lists in table 10 and are referred to as "Relative position". In those three constituencies with 2 adjustment seats the relative position of the second best candidate of each list is also shown.

² However, even if the law would have allowed these two candidacies to take part in the allocation process they would not have won any of the adjustment seats.

<i>Table 10:</i>		<i>Relative position of the candidates for adjustment seats in 2003</i>				
<i>Party symbol:</i>		B	D	F	S	U
Northwest		7,23%	7,39%	7,12%	7,74%	5,31%
Northeast		6,55%	7,84%	5,64%	7,78%	7,06%
South		7,90%	7,30%	4,37%	7,42%	4,66%
Southwest	First	7,46%	7,68%	6,75%	6,55%	6,24%
	Second	4,97%	6,40%	3,37%	5,46%	3,12%
Reykjavik South	First	5,67%	7,61%	6,64%	8,33%	4,66%
	Second	3,78%	6,34%	3,32%	6,66%	3,11%
Reykjavik North	First	5,81%	8,88%	5,54%	7,25%	4,89%
	Second	3,87%	7,10%	2,77%	6,05%	3,26%

Let us have a look at these calculations for the Liberal Party (F) in the Northwest District. According to table 3 it won 2666 votes in that constituency and has already received 1 constituency seat. Thus d'Hondt's quotient of its next candidate in a continuation of the constituency apportionment would be $2666/2 = 1333$. This number has to be seen relatively with respect to the total number of eligible votes cast in the constituency which is 18710; see again table 3. The crucial outcome is therefore $1333/18710 = 7.12\%$

According to the column for the F-list in table 10 the relative position of the F-party is highest in the Northwest. Therefore, as the F-party is eligible for the first adjustment seat according to table 10, that seat is allocated to this list of that party. Note that there are three stronger candidates in this constituency, but they belong to other parties. The national outcome dictates that the F-party should receive this seat as its first adjustment seat.³ Simultaneously the Northwest District has now received its one and only adjustment seat. Therefore that constituency will not be taken further into account when the relative position of party lists are considered.

Table 11 shows this allocation process of the 9 adjustment seats in total.

³ The current Norwegian election law is almost identical to the Icelandic one, except for this item. In Norway the first step of the allocation process of the adjustment seats – i.e. the calculation of the “national ranking numbers” and the corresponding allocation of the seats to the parties on a nationwide basis – is carried out first. The second step – the allocation of the adjustment seats to individual party lists – is (if applied to Iceland) based on the relative highest position of the lists nationally, not respecting the ranking of the national numbers as in current Icelandic law. According to the Norwegian election law the first adjustment seat would go to the D-party in Reykjavik North as it has the highest relative position among all lists in the country, 8.88%, see table 10.

Seat No.	"National ranking numbers"	Highest relative position	Party list	Constituency where allocated
1	4 508	7.12%	F	Northwest
2	3 381	6.75%	F	Southwest
3	3 226	7.06%	U	Northeast
4	3 085	8.88%	D	Reykjavik North
5	2 984	8.33%	S	Reykjavik South
6	2 938	7.68%	D	Southwest
7	2 835	7.42%	S	South
8	2 805	7.61%	D	Reykjavik South
9	2 707	5.81%	B	Reykjavik North

Until the sixth seat is allocated the candidate of the corresponding party with the highest relative position is selected. In allocating the 6th seat the “national ranking numbers” demand that it shall go to party D. The list of that party with the highest relative position is in the Northeast (7.84%), but the only adjustment seat of that constituency has already been taken. The highest relative position in those constituencies still with vacant seats is in the Southwest constituency (7.68%). Therefore this allocation takes place there.

In table 12 this allocation of adjustment seats is shown in total and finally the combined apportionment of constituency and adjustment seats in the 2003-elections is shown in table 13.

<i>Party symbol:</i>	B	D	F	S	U	Totals
Northwest	-	-	1	-	-	1
Northeast	-	-	-	-	1	1
South	-	-	-	1	-	1
Southwest	-	1	1	-	-	2
Reykjavik South	-	1	-	1	-	2
Reykjavik North	1	1	-	-	-	2
Eqalization seats in total	1	3	2	2	1	9

Table 13 shows the combined results of tables 8 and 12.

<i>Party symbol:</i>	B	D	F	N	S	T	U	Totals
Northwest	2	3	2	-	2	-	1	10
Northeast	4	2	-	-	2	-	2	10
South	2	3	1	-	4	-	-	10
Southwest	1	5	1	-	4	-	-	11
Reykjavik South	1	5	-	-	4	-	1	11
Reykjavik North	2	4	-	-	4	-	1	11
Seats in total	12	22	4	-	20	-	5	63

It should be noted that the final national assignment of seats (last row in table 13) is a proportionally proper allocation of the seats, i.e. if all 63 seats were allocated to the parties on the basis of the sum of votes from all the constituencies (applying d’Hondt’s method) the allocation would be the same.

9. *Apportionment of constituency seats in the 2007-elections*

Only the outcome, not the calculations, of the allocation steps will be shown in the case of the 2007-elections.

<i>Table 14: Apportionment of constituency seats in total in 2007</i>							
<i>Party symbol:</i>	B	D	F	I	S	V	Totals
Northwest	1	3	1	-	2	1	8
Northeast	2	3	-	-	2	2	9
South	2	4	-	-	2	1	9
Southwest	1	5	-	-	3	1	10
Reykjavik South	-	5	-	-	3	1	9
Reykjavik North	-	4	-	-	3	2	9
Constituency seats in total	6	24	1	-	15	8	54

<i>Table 15: Apportionment of adjustment seats in total in 2007</i>						
<i>Party symbol:</i>	B	D	F	S	V	Totals
Northwest	-	-	1	-	-	1
Northeast	1	-	-	-	-	1
South	-	-	1	-	-	1
Southwest	-	1	-	1	-	2
Reykjavik South	-	-	1	-	1	2
Reykjavik North	-	-	-	2	-	2
Compensatory seats in total	1	1	3	3	1	9

<i>Table 16: Apportionment of seats in total in 2007</i>							
<i>Party symbol:</i>	B	D	F	I	S	V	Totals
Northwest	1	3	2	-	2	1	9
Northeast	3	3	-	-	2	2	10
South	2	4	1	-	2	1	10
Southwest	1	6	-	-	4	1	12
Reykjavik South	-	5	1	-	3	2	11
Reykjavik North	-	4	-	-	5	2	11
Seats in total	7	25	4	-	18	9	63

As in the 2003-elections the final allocation of all 63 seats is proportionally fair given the hindrance of the 5%-threshold which excludes the party I from sharing adjustment seats.

10. *Assignment of seats to individual candidates, illustrated by the 2007-outcome*

As mentioned in the introduction, section 2, lists are nominated in ranked order by the candidature. However, the electors have the right to alter this order by renumbering individual candidates or to cross out those that they do not like. How this can be done is illustrated by an (constructed) example in the left half of table 17.

<i>Table 17: Example of a changed ballot</i>					
Nomination order	Re-ordering by the elector	Name of candidate. The elector has crossed out one candidate	New ranking after changes made by the elector	Share of a vote	
				If no re-ranking	After re-ranking
1		<i>Jón Jónsson</i>	2	1.00	0.75
2		<i>Anna Sigurðardóttir</i>		0.75	0.00
3	1	<i>Guðrún Magnúsdóttir</i>	1	0.50	1.00
4		<i>Pétur Guðmundsson</i>	3	0.25	0.50
5		<i>Sigríður Björnsdóttir</i>	Candidates in places 5, 6 etc. are ignored (see text)		
6	2	<i>Magnús Jónsson</i>			
...		...			

The changes made by the elector apply only to the candidates in first places on the list equal to twice the number of seats allocated to the list (but with a minimum of three). It is assumed in the example of table 17 that the particular list has won 2 seats, hence the first 4 candidates are affected by the manipulations of the elector. The elector's re-ranking of the candidate in place 6 is thus ignored. The ranking of these top 4 candidates are evaluated on the basis of the *Borda*-method. The top candidate is assigned a value of 1 full so-called *personal vote*, the next one 1/4 less or 0.75 of a vote etc., since the ranking applies to 4 candidates. The personal votes of these candidates are shown in the two last columns of table 17, first as if the elector would not have made any changes on the ballot. The last column shows these vote-values taking care of the changes made by the elector. Note in particular that the candidate that the elector crossed out gets a zero vote.

The personal votes thus calculated are summed up for each of the eligible candidates on the list and is the basis for the assignment of the seats acquired by the list to these candidates.

Unfortunately the only effective way for the electors to change the order of candidates is to use the "negative" method of crossing out candidates. Table 18 shows how a united group of electors can achieve that a candidate is re-ranked up one place by crossing out the candidate above him and placing the favored candidate in place 1. The minimum size of this group needed to lift a candidate up one step depends on the number of seats acquired by the list and ranges from 25% down to 7.7% if the list has won 6 seats.

<i>Table 18: Minimum relative size of a group of electors of a particular list needed to lift a candidate up one place by crossing out the one above him and ranking the favored candidate as no. 1.</i>	
Number of seats allocated to the list	Portion of the voters of the list
1	25,0%
2	20,0%
3	14,3%
4	11,1%
5	9,1%
6	7,7%

In the elections 2007 changes by the electors made a difference for the first time since 1946. It should be noted however that in the period 1959-2000 the rules of calculating the personal votes were different making it very difficult for the electors to have any influence.

In the 2007-elections two candidates (incidentally) from the same party but in different constituencies dropped down one place on their lists due to ballot changes (mainly by being crossed out). However, neither of them lost their seat in the Parliament.

Table 19 shows these personal votes in the South constituency for the nine constituency seats. Candidate no. 3 on the list of the party with the symbol D was lifted up to place 2 whereas the candidate ranked by the party in place 2 dropped down to place 3. The party, i.e. its list, got 4 seats so no harm was done!

<i>Table 19: Personal votes of candidates in the South constituency in 2007 with and without changes made by the electors</i>					
Allocation order of seats	Name of candidate	Party symbol	Nomination order on the lists	Personal votes taking care of changes made by the electors	Personal votes with these changes being ignored
1	Árni M. Mathiesen	D	1	8904	9120
2	Björgvin G. Sigurðsson	S	1	6737	6783
3	Guðni Ágústsson	B	1	4700	4745
4	Kjartan Þ. Ólafsson	D	3	7054	6840
5	Lúðvík Bergvinsson	S	2	4958	5087
6	Árni Johnsen	D	2	6284	7980
7	Atli Gíslason	V	1	2493	2498
8	Bjarni Harðarson	B	2	3554	3559
9	Björk Guðjónsdóttir	D	4	5965	5700

11. On the quality of the apportionment

The Icelandic election system is an assignment problem with two sets of constraints:

1. **Constituency constraints:** Prior to the election it is known how many seats are to be assigned within each of the 6 constituencies. This is the first set of constraints.
2. **Party constraints:** Article 31 of the Icelandic Constitution states one of the main objectives of any election law as follows: “[Adjustment] seats ... shall be distributed ... with the purpose of providing individual political parties with representation reflecting to the fullest possible extent each party’s total [i.e. national] number of votes ... [however, respecting the 5% threshold].” This effectively means that the sum of seats allocated to each party is dictated by the national outcome of the election. This constitutes the other dimension of the sets of constraints.

Given these constraints the problem facing the lawmakers is to construct a method to allocate seats to the individual lists (generally one list per party in each constituency) in such a way that the two sets of sums mentioned above are respected. But, of course, at the same time the assignment must in some sense be proportional to the votes of the individual lists.

This is mathematically a challenging problem. Balinski and Demange have set up axioms for such apportionment methods.⁴ These axioms are from a democratic point of view self evident as minimum requirements. Balinski and Demange nevertheless

⁴ “An axiomatic approach to proportionality between matrices”; ML Balinski, G Demange - Mathematics of Operations Research, 1989, Vol. 14, No. 4, November 1989.

prove that there is only one solution, only one method, satisfying these axioms or requirements, given that the apportionment is to be based on a specific so-called divisor method, e.g. d'Hondt's rule.

This unique optimal method can be easily formulated in the case d'Hondt's rule is used as a basis. The solution is found by maximizing the product of votes per seat. Technically the solution can be found with a standard spreadsheet package like Excel.⁵

Table 20 shows the differences between the official and the optimal allocation of seats in the 2003-elections. The difference is minimal as any two different allocations can never involve less than 4 lists or cells.

<i>Table 20: Deviations of the official apportionment from the optimal one in 2003: +1 (-1) means that a seat should be added to (subtracted from) the official allocation to get the optimal one.</i>								
<i>Party symbol:</i>	B	D	F	N	S	T	U	Sums
Northwest	2	3	2	-	2	-	1	10
Northeast	4	2	-	-	2	-	2	10
South	2+1	3	1	-	4-1	-	-	10
Southwest	1	5	1	-	4	-	-	11
Reykjavik South	1	5	-	-	4	-	1	11
Reykjavik North	2-1	4	-	-	4+1	-	1	11
Seats in total	12	22	4	-	20	-	5	63

In table 21 it is shown why the official allocation is improved (and indeed made optimal) by the four changes shown in table 20. The product of votes per seat is increased by 42% by the changes, showing at least an improvement, although more mathematics is needed to prove that these changes make it the optimal solution.

<i>Table 21: Changes in votes per seat by the changes shown in table 20.</i>					
<i>The colored cells show votes per the last seat of the lists. Green cells refer to lists with one seat too many in the official allocation whereas the orange ones refer to the opposite situation.</i>					
	Official allocation		Optimal allocation		Increase of the product of votes per seat
	B	S	B	S	
South	2 967	1 857	1 978	2 475	
Reykjavik North	2 100	3 278	4 199	2 622	
Product of the four relevant	37 902 781 266 699		53 906 177 801 528		42%

In the 2007 elections the official allocation of seats was further away from the optimal allocation than was the case in 2003 as can be seen in table 22.

⁵ The task can be formulated as a linear programming problem; see also footnote 1. Due to the so-called unimodularity of the constraint sets the solution is always a proper integer solution. The objective function, i.e. the product, can be made linear by taking the logarithm of the product.

<i>Table 22: Deviations of the official apportionment from the optimal one in 2007: +1 (-1) means that a seat should be added to (subtracted from) the official allocation to get the optimal one.</i>							
<i>Party symbol:</i>	B	D	F	I	S	V	Totals
Northwest	1+1	3	2-1	-	2	1	9
Northeast	3	3	0	-	2	2	10
South	2	4	1-1	-	2+1	1	10
Southwest	1-1	6	0+1	-	4	1	12
Reykjavik South	0	5	1	-	3	2	11
Reykjavik North	0	4	0+1	-	5-1	2	11
Seats in total	7	25	4	-	18	9	63

It should be noted that the optimal assignment for the elections in 2007 calls for changes of two constituency seats, not only a reshuffling of the adjustment seats. This would not be allowed given the current constitutional framework for an election act. The changes that would require a deviation from this framework are indicated with red figures in table 22.

Mathematical Formulation of the Apportionment of Seats

12. Terminology

Let SCC_i be the number of constituency seats in constituency i and SEC_i be the number of adjustment seats attributed to the same constituency.

The following is determined by the election act (numbers are partly specific for the elections in 2003):

$$(1) \quad SCC_i = 9 \text{ for all } i \\ \text{and } SEC_i = 1 \text{ for } i=1,2,3 \text{ and } SEC_i=2 \text{ for } i=4,5,6.$$

Let V_{ij} be the number of votes cast for list j in constituency i .

Furthermore let VC_i be the total number of valid votes cast in constituency i and VP_j be the total number of votes cast for party j and VN the grand total of the number of valid votes cast nationally.

Obviously the following holds:

$$(2) \quad \sum_j V_{ij} = VC_i, \sum_i V_{ij} = VP_j \text{ and } \sum_j VP_j = \sum_i VC_i = VN.$$

Let x_{ij} be the number of seats to be apportioned to party j as constituency seats in constituency i and y_{ij} the corresponding number of adjustment seats. The apportionment has to respect the following constraints:

$$(3) \quad \sum_j x_{ij} = SCC_i \text{ and } \sum_j y_{ij} = SEC_i.$$

Later we will refer to the following sums

$$(4) \quad Cx_j = \sum_i x_{ij}$$

i.e. the number of constituency seats apportioned to party j in total.

13. Apportionment of constituency seats

The constituency seats are apportioned on the basis of the d'Hondt quotients:

$$(5) \quad V_{ij}/d \text{ where } d = 1, 2, 3, \dots$$

The seats are allocated within constituency i by picking the SCC_i largest quotients. This allocation yields values to x_{ij} .

14. Allocation of adjustment seats

First it must be decided which parties are eligible for taking part in the allocation of adjustment seats. These are the parties that exceed a 5% threshold on a national basis, i.e. that fulfill the requirement:

$$(6) \quad VP_j \geq 0.05 VN.$$

In the following, when referring to a party with index j , we will omit those parties that do not fulfill requirement (6).

Next the so-called national ranking numbers for those parties that are eligible according to (6) have to be calculated as follows:

$$(7) \quad VP_j / (Cx_j + d) \text{ where } d = 1, 2, 3, \dots$$

The 9 largest of these are chosen. Let j_1, j_2, \dots, j_9 be the indices of these 9 largest national ranking numbers.

For later use we have to calculate the following reference numbers:

$$(8) \quad \frac{V_{ij} / (x_{ij} + d)}{VC_j} \text{ with } d = 1 \text{ for } i = 1, 2, 3 \text{ and } d = 1 \text{ and } 2 \text{ for } i = 4, 5, 6.$$

The first adjustment seat is allocated to party j_1 . To find out which of the lists of party j_1 should receive this seat the constituency with the highest reference number, see (8), must be identified. Label this constituency as i_1 . Thus the first adjustment seat goes to the list of party j_1 in constituency i_1 . This process now continues by finding the highest reference number for party j_2 taking notice of the fact that one (and possibly the only one) adjustment seat has already been allocated in constituency i_1 .

This is continued until the last seat is allocated to party j_9 in the only constituency that has one adjustment seat missing.^{6 7}

⁶ It may in exceptional cases happen that a party is not represented in any of those constituencies which still have vacant adjustment seats. In this case that party will not get allocated any further seats and the list of national numbers is extended as needed to replace the vacancy caused by the exclusion of the particular party.

⁷ The previously mentioned Norwegian election law differs from the Icelandic one in the following items:

- It is based on Sainte-Laguë's rule instead of that of d'Hondt.
- The "national ranking numbers" are only used to distribute the adjustment seats among the parties; not for governing the order of which these seats are allocated to the party lists as is the case in Iceland.
- The reference numbers (8) are in Norway $\frac{V_{ij} / (2x_{ij} + 1)}{VC_i / SCC_i}$. The numerator, although looking

different, is principally the same as in the Icelandic system. In Norway it is the Sainte-Laguë quotient for the next candidate following those that have already received a constituency seat. The denominator however is not the total number of votes as in Iceland but instead the

15. Assignment of seats to individual candidates

Let us consider a particular constituency list which has won S seats (constituency and/or adjustment seats). Let us call R the ranking number of the list. It is defined as

$$(9) \quad R = \max(3; 2S)$$

Let $a_{ijk} = 1$ if on the k -th ballot of the list a candidate nominated for seat i is placed directly or indirectly in seat j . Otherwise let $a_{ijk} = 0$. In this connection let a crossing out of a seat be reflected as assigning $a_{ijk} = 1$ to seat number $j = R+1$.

The sum of personal votes of candidate i is calculated as

$$(10) \quad P_i = \sum_k \sum_{j=1}^{R+1} a_{ijk} (R+1-j) / R$$

These personal votes in descending order govern the final order of the candidates of the list.

average number of votes behind each constituency seat. As the number of constituency seats was the same in all constituencies in Iceland in the 2003-elections this would not have made a difference in the Icelandic case; see also footnote 1.

- Finally the adjustment seats are allocated to the constituency lists on the basis of the reference numbers; the national ranking numbers have no influence here, as said before.